Designing a 5th Order Low-pass Chebyshev Filter

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ANALOG ELECTRONIC SYSTEMS (ECCS 394)

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INTRODUCTION & KEY NOTIONS

Analog filters handle continuous time signals, they are widely used in applications where their digital counterparts fail to perform, most important in:

+ Huge amplitude dynamic range.
+ Speed.
+ Huge frequency dynamic range.

When designing such filters, some parameters need to be taken in consideration, i.e. the cutoff frequency, order or minimum attenuation at desired frequency.

Electronic filters must be divided in two main branches, active or passive, then they can be categorized by topology (if active), type (behavior) and order.

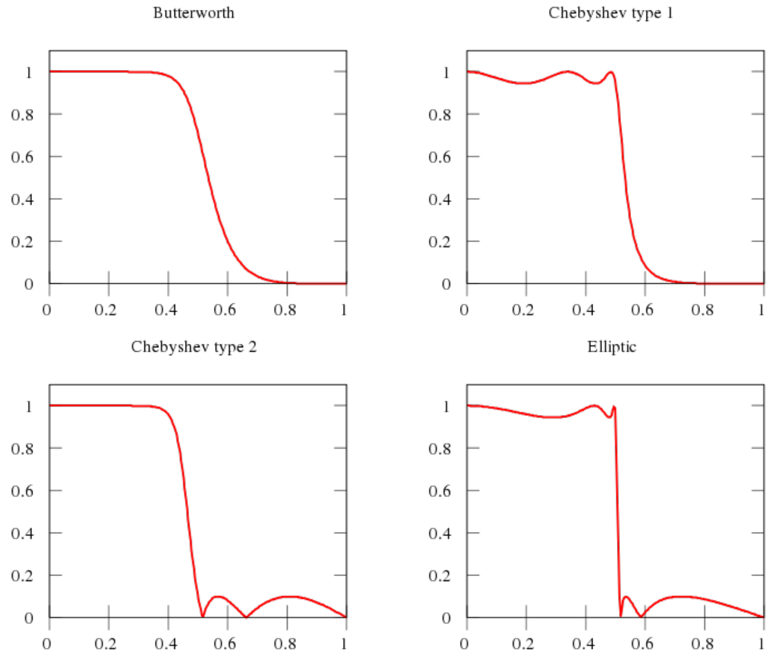

Fig. 1: Comparative between Butterworth, Chebyshev and Elliptic filters illustrated by fifth-order low-pass filters. This is a demonstration of behavior.

The topology mainly describes the method the circuit will be built physically by arranging electronic components in particular ways, type behavior (Fig. 1) describes the frequency response or time domain characteristics of the filter and the order primary depends on the transfer function. For example, one can build a 5^{th} order Chebyshev or Butterworth filter with a Sallen-Key topology, while someone else may choose to implement the same on a different topology such as multiple feedback.

DESCRIPTION

The current design in this paper describes and explains the procedure to design a 5^{th} order low-pass Chebyshev filter in two different active topologies; multiple feedback and Sallen-Key, the objective of this is to compare the frequency and time-domain behavior and make sure the design works as expected.

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DESIGN

The 5th order Chebyshev filter requires 3 stages in the **Sallen-Key** configuration, the normalized table for a 3 dB ripple is the following:

Stage	ai	bi	Qi
1st	5.6334	0.0000	-
2nd	0.7620	2.6530	2.1375
3rd	0.1172	1.0686	8.8178

According to this table, the transfer function is shown in equation (1).

$$H(s)=\frac{1}{b_1 s^2+a_1 s+1} \qquad (1)$$

where for a Sallen-Key configuration with unity gain;

$$b_i=C_1 C_2 R_2 R_3 \quad , \quad \omega_o=\frac{1}{\sqrt{b_i}}=\frac{1}{\sqrt{C_1 C_2 R_2 R_3}} \quad \text{or}$$

$$f_o=\frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \quad \text{and} \quad Q=\frac{\sqrt{R_1 R_2 C_1 C_2}}{(R_1+R_2)C_2}$$

The transfer function for the **first** stage is simplified as (2):

$$H(s)=\frac{1}{a_1 s+1} \qquad (2)$$

Which means it will be implemented as a first order active-filter (Fig. 2).

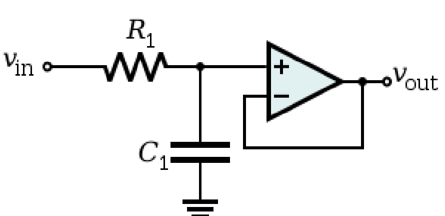

Fig. 2: 1st Order active filter.

And the **second** and **third** stages have the form of equation (1), therefore it will be implemented as Fig. 3.

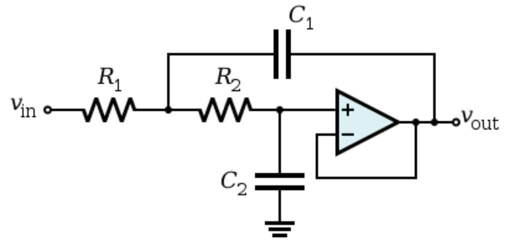

Fig. 3: 2nd Order Sallen-Key.

Now, to calculate the values for the **first** stage:

Since $a_i=2\pi R_1 C_1$, by taking a_i from the normalized table and defining $C_1=200[\mu F]$ then $R_1=4.48\,k\Omega$.

Then, we calculate the values or the **second** stage:

$$C_1=1 \quad , \quad C_2=n \quad , \quad R_1=R \quad , \quad R_2=mR \quad .$$

then $\omega_o=0.6139$ and $m=\frac{1}{2\pi\,\omega_o QR}-1$.

If we define $R=0.1$ then $m=0.2115$ and $n=\frac{1}{m}\left(\frac{1}{2\pi R\omega_o}\right)^2=31.78$.

And for the **third** and final stage:

$$C_1=1 \quad , \quad C_2=n \quad , \quad R_1=R \quad , \quad R_2=mR \quad .$$

then $\omega_o=0.9674$ and $m=\frac{1}{2\pi\,\omega_o QR}-1$.

If we define $R=0.01$ then $m=0.8653$ and $n=\frac{1}{m}\left(\frac{1}{2\pi R\omega_o}\right)^2=312.8$.

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Next, the summary of our normalized to 1 Hz values.

1st Stage			
R_1	4.48 kΩ	-	-
C_1	200 uF	-	-

2nd Stage			
R_1	0.1 Ω	R_2	0.02115 Ω
C_1	1 F	C_2	31.78 F

3rd Stage			
R_1	0.01 Ω	R_2	0.008653 Ω
C_1	1 F	C_2	312.8 F

Then it is scaled in frequency and impedance to get the next table:

1st Stage			
R_1	4.48 kΩ	-	-
C_1	10 nF	-	-

2nd Stage			
R_1	4.7 kΩ	R_2	994 Ω
C_1	1.06 nF	C_2	33.8 nF

3rd Stage			
R_1	470 Ω	R_2	407 Ω
C_1	1.06 nF	C_2	333 nF

This scaling yields to pretty close commercial values.

The following implementation uses mostly commercial resistor values.

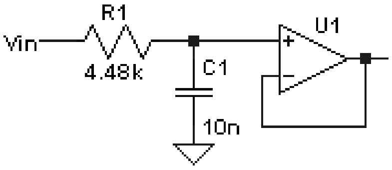

Fig. 4: 1st Active low-pass stage.

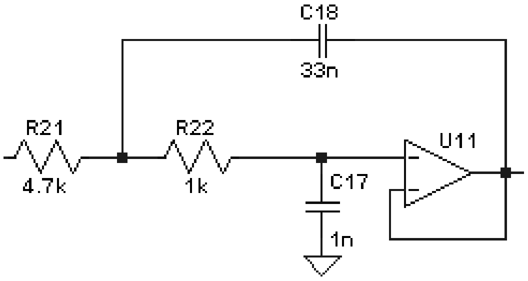

Fig. 5: 2nd Sallen-Key stage.

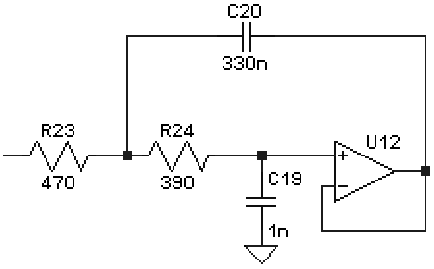

Fig. 6: 3rd Sallen-Key stage.

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Now to design the same filter but in multi-feedback topology.

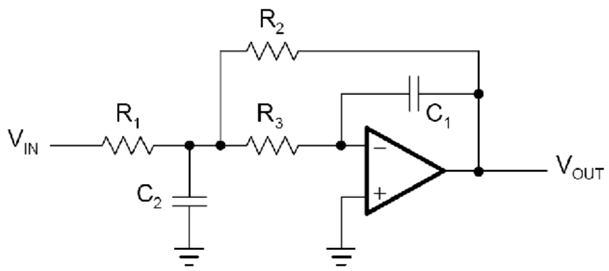

Fig. 7: Multiple feedback configuration.

And relationship of components in a multiple-feedback configuration is;

$$b_i = C_1 C_2 R_2 R_3 \quad , \quad \omega_o = \frac{1}{\sqrt{b_i}} = \frac{1}{\sqrt{C_1 C_2 R_2 R_3}} \quad \text{or}$$

$$f_o = \frac{1}{2\pi \sqrt{R_1 R_2 C_1 C_2}} \quad \text{and} \quad Q = \frac{\sqrt{R_2 R_3 C_1 C_2}}{(R_1 + 2R_3) C_1}$$

due for a unity gain requirement, $R_1 = R_2$.

The first stage of the active low-pass filter will be the same, and the **second** stage calculations are:

$$C_1 = 1 \quad , \quad C_2 = n \quad , \quad R_1 = R \quad , \quad R_2 = mR \quad .$$

then $\omega_o = 0.6139$ and $m = \frac{1}{4\pi \omega_o Q R} - \frac{1}{2}$.

If we define $R = 0.1$ then $m = 0.1056$ and

$$n = \frac{1}{m} \left(\frac{1}{2\pi R \omega_o} \right)^2 = 63.6 \quad .$$

Then for the **third** and last stage:

$$C_1 = 1 \quad , \quad C_2 = n \quad , \quad R_1 = R \quad , \quad R_2 = mR \quad .$$

then $\omega_o = 0.9674$ and $m = \frac{1}{4\pi \omega_o Q R} - \frac{1}{2}$.

If we define $R = 0.01$ then $m = 0.4327$ and

$$n = \frac{1}{m} \left(\frac{1}{2\pi R \omega_o} \right)^2 = 625.56 \quad .$$

Next, the summary of our normalized to 1 Hz values.

1st Stage			
R_1	4.48 kΩ	-	-
C_1	200 uF	-	-

2nd Stage			
$R_1 = R_2$	0.1 Ω	R_3	0.01056 Ω
C_1	1 F	C_2	63.6 F

3rd Stage			
$R_1 = R_2$	0.01 Ω	R_3	0.004327 Ω
C_1	1 F	C_2	625.56 F

Then it is scaled in frequency and impedance to get the next table:

1st Stage			
R_1	4.48 kΩ	-	-
C_1	10 nF	-	-

2nd Stage			
$R_1 = R_2$	1 kΩ	R_3	105.6 Ω
C_1	5 nF	C_2	318 nF

3rd Stage			
$R_1 = R_2$	1 kΩ	R_3	432.7 Ω
C_1	500 pF	C_2	312.78 nF

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The implementation of the multi-feedback configuration is shown in the following figures.

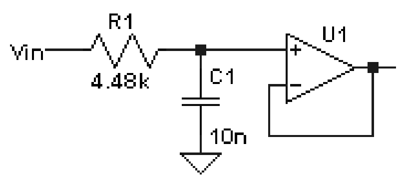

Fig. 8: 1st Active low-pass stage.

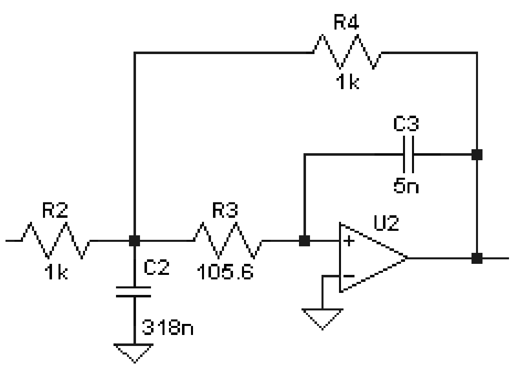

Fig. 9: 2nd stage for multi-feedback.

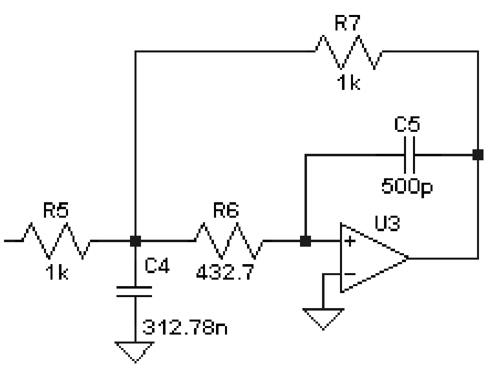

Fig. 10: 3rd stage for multi-feedback.

SIMULATIONS

First, we will analyze the frequency response of the **Sallen-Key** (Fig. 11) configuration with a cutoff frequency of 20 kHz..

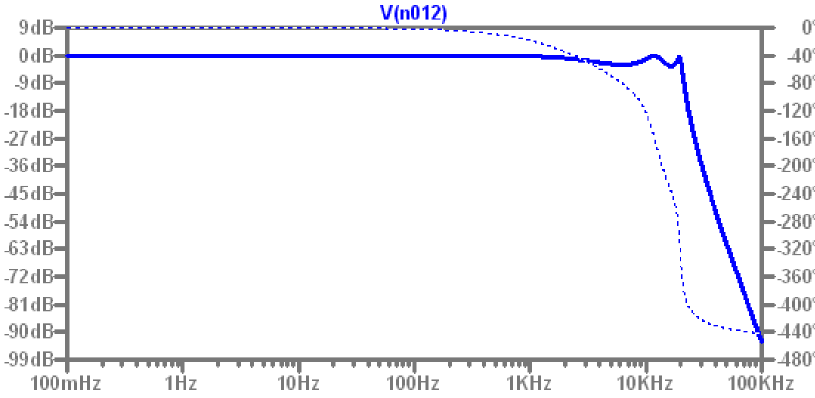

Fig. 11: 5th Order Chebyshev on a Sallen-key configuration.

Fig. 12 shows the differences between the ideal component values on the simulations vs the commercial values implementation.

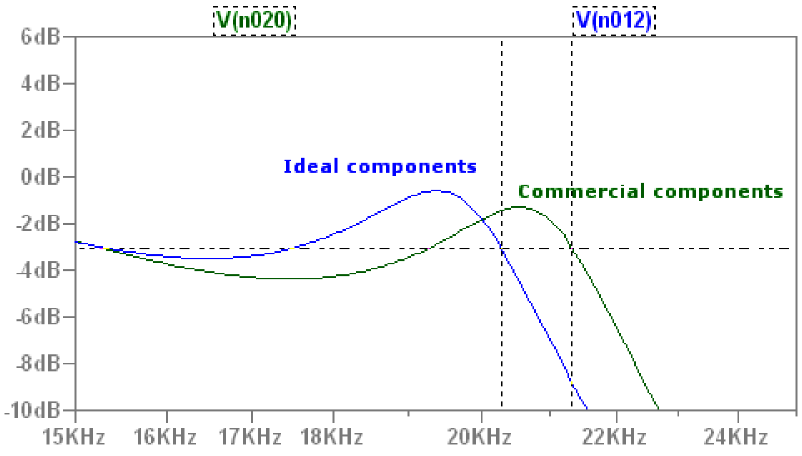

Fig. 12: Comparative: Ideal vs. Commercial implementation.

We can see a slight shift in the cutoff frequency (-3 dB positioned with the marker) and a slight increase in the ripple amplitude.

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In Fig. 12 & Fig. 13 we can check the differences between the real design and the implemented filter.

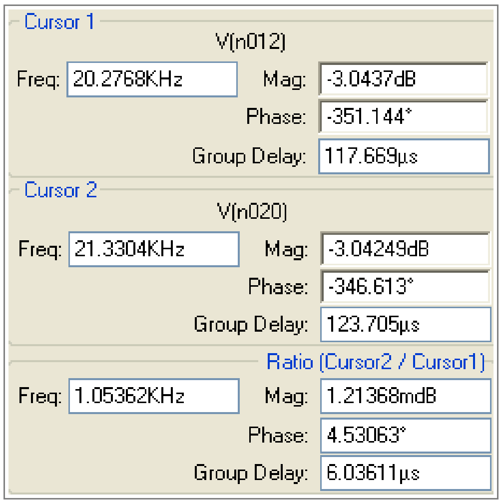


Fig. 13: Dialog box displaying the cursor markers from Fig. 10.

Next, we will be analyzing and comparing the same (ideal) Chebyshev filter but on two different topologies:

a) Sallen-Key.
b) Multiple feedback.

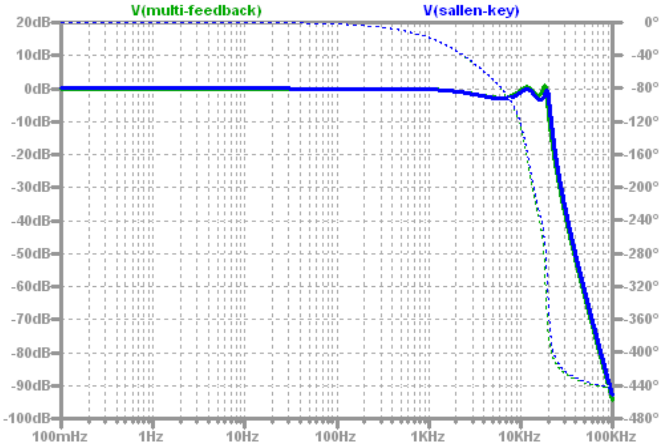
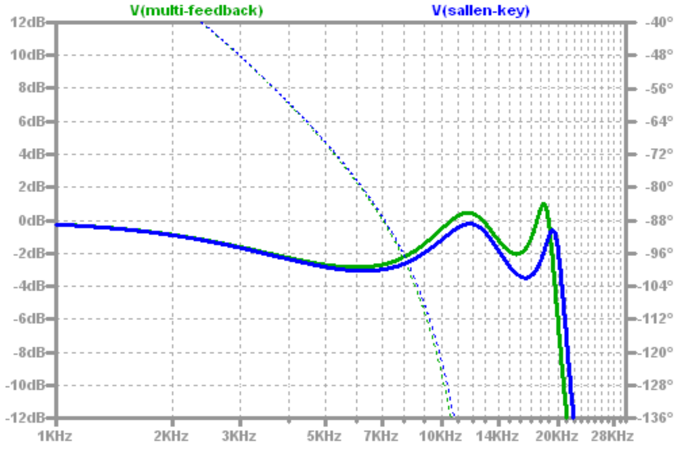


Fig. 14: Up: Freq. Response comparative between topologies. Down: Zoom in the ripple



As It is seen in the frequency response comparison, the difference between multi-feedback and Sallen-Key is minimal because the phase is approximately the same during the whole spectrum, and only slight differences in the magnitude are seen near the ripple and f_C .

After building the circuit and plotting a table, we can then plot the expected frequency response (Fig. 15).

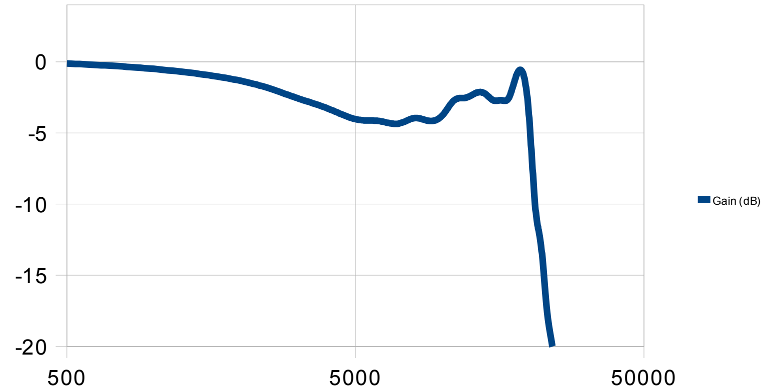


Fig. 15: Measured frequency response.

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CONCLUSIONS

Analog filters offer lots of different design implementations, we could not notice big differences between behavior between different topology implementations. One thing to take into consideration is the number of components, for example the multi-feedback topology has one more component (resistor in the low-pass case) than the Sallen-Key.

Of course that filters offer great functionality in everyday applications such as radio tuning, noise filtering , band suppression. The already mentioned advantages over digital filters also make them important for communication appliances or radio frequency.

REFERENCES

[1] R.C. Jager, Microelectronic Circuit Design, New York, NY: Mc-Graw Hill, 1997.

[2] Ron Mancini, Op Amps For Everyone, Texas Instruments, August 2002.

[3] A. S. Sedra and K. C Smith, Microelectronic Circuits. New York, NY: Oxford University Press, 2003.

[4] R. Jacob Baker, CMOS Circuit Design, Layout, and Simulation,New York, Wiley Interscience, 1997.